‘If I have seen further it is by standing on the shoulders of giants.’
Isaac Newton

Foundational Research of Math Education at Happy Numbers

HappyNumbers.com

Introduction

HappyNumbers.com teaches students to "think math" by exploring the meaning behind the math, building upon simple concepts to create connections and develop deep understanding. We help K-2 teachers differentiate instruction and deepen students' conceptual understanding of math. Happy Numbers has been built upon a firm foundation of research findings and best practices in education. Driven by pedagogy and research, we don't just agree with the ideas of great educators and psychologists, we "stand on the shoulders" of these giants and put their ideas into action as a practical teaching tool.

In doing so, our essential practices are:

- Differentiating instruction
- Promoting conceptual understanding along with procedural efficiency
- Minding the mindset
- Supporting English Language Learners

Differentiating Instruction

Research shows that when students are given adequate time, attention, and help (as in a 1:1 tutoring session), they achieve far greater results than in a traditional educational model. Through a mastery learning approach, researchers have found a "2 sigma effect," in which about 90% of the tutored students performed at the level of the top 20% of conventionally taught students (Bloom, 1968, 1974, 1984). While providing 1:1 instruction and support for each student would have obvious benefits, the logistical drawbacks are prohibitive. Instead, teachers can use technology to simulate individualized instruction more and more adequately.

Happy Numbers functions as a "teacher assistant" that helps classroom teachers adapt instruction to the individual needs of students. Not

only do students receive differentiated instruction while engaged with Happy Numbers, but they also receive more individualized attention as teachers are freed up to work with small groups.

To differentiate learning, Happy Numbers implements an approach based on the Zone of Proximal Development, or ZPD (Vygotsky, 1934). This approach suggests looking for a zone between problems that a student can solve on his own and those that he cannot solve. Providing appropriate guidance to help students solve problems within this ZPD is an effective way to develop proficiency. In other words, guidance in the ZPD helps advance the zone, leaving the newly mastered problems in the proficiency zone.

Happy Numbers uses technology to find each student's individual ZPD for a certain skill. First, we interactively guide students through an introduction and the easiest problems related to that skill. Students who can easily solve problems at this level are in their proficiency zone, so we immediately move them to the next level. Happy Numbers provides all necessary scaffolding to students working in their ZPD. Our levels are fine-tuned, so that advancing from one to the next is a smooth transition.

While a student works within his ZPD, we individualize the scaffolding based on that student's interaction with the system. Our technology allows us to do this in several ways:

- providing appropriate textual hints
- displaying visual feedback
- introducing interactive manipulatives
- adjusting numerical or graphical data in problem statements
- increasing or reducing the number of exercises

Thoughtfully weighed, the system response is just enough to support student mastery before moving ahead. Happy Numbers feedback (hints, visual support, adjusted exercises, positive reinforcement) is highly individualized. Equally important is the timing of such feedback — as an instant response to a student's actions. At the same time, we avoid over-instruction to keep motivation and interest high.

The effectiveness of this scaffolding approach is rooted in the psychology of learning (Skinner, 1968) and confirmed by research in

methods of teaching math, such as using a developmental progression when teaching numbers and operations as well as geometry and other domains (Frye et al., 2013).

In the larger scheme, Happy Numbers also supports student learning through a carefully-scaffolded overall progression of exercises. Curriculum sequencing is the art of balancing progress with proficiency. We sequence our topics

- to ensure timely mastery of the prerequisites
- to space learning of a certain concept or skill over time
- to avoid monotony

Spaced learning over time is more effective than massive learning, as many teachers know from their experience and as confirmed by direct research, especially in retaining knowledge (Rogher et el, 2006; Son et al. 2012).

Conceptual Understanding via Multiple Representations and Strategies

The benefits of conceptual understanding are tremendous, with the primary benefit being the ability to apply knowledge to new situations. For example, if a student understands that 10 ones can be exchanged for 1 ten (and back), and likewise 10 tens for 1 hundred, he will more easily master addition and subtraction of 3-, 4-,… n-digit numbers.

Students in primary grades are at an important turning point — the beginning of logical or operational thought (Piaget, 1936). This means they can comprehend and apply mathematical concepts, with appropriate support and at the level corresponding to their age.

The **concrete-pictorial-abstract approach** (Bruner, 1964) is the foundation on which Happy Numbers builds its interaction with

students to initialize and deepen understanding of math concepts:

- Virtual manipulatives are almost real objects, colorful and attractive. We use a variety of them and we use them a lot. Young students easily operate virtual manipulatives, with fun that reinforces comprehension.
- Pictorial representations are often animated, which makes them more expressive, explanatory, and memorable.

Multiple representations are a powerful tool we use to help students understand a math concept, relation, or fact (Ainsworth, 1999; Ball et al., 2008). Happy Numbers implements a variety of representations. For example, we represent addition and subtraction with:

- concrete manipulatives: virtual objects such as birds or balloons
- more abstract manipulatives: base-10 blocks or place value cards
- pictures
- animations of putting together/taking away
- a number line
- a hundred chart
- number bond diagrams
- equations

By presenting concepts across different representations we ensure cognitive connections that lead to mathematical flexibility. A student's ability to move between several representations of a math concept is a strong indication that he fully understands the concept (Lesh et al., 1983).

Connecting concepts rather than teaching them as stand-alone is our firm position. Happy Numbers takes this task seriously: primary grade math is no longer the "arithmetic" of a century ago. In order to facilitate understanding across concepts, we always show connections. For example, we have students work with multiple interpretations of whole numbers, such as:

- counting sequence
- cardinality of a set
- "names" of points on number line
- results of measurement (related to the unit used)
- results of operations

By incorporating these best pedagogical practices, we ensure that

students gain a deep understanding of math concepts and strong procedural fluency.

Growth Mindset

Praising the process through which success was achieved helps students develop the "growth mindset" and the belief that effort makes them stronger, which leads to higher achievement (Dweck, 2006). With Happy Numbers, students are intrinsically motivated by this self-confidence. Therefore, they work hard to progress through the exercises. Any errors along the way are positioned as opportunities to learn rather than mistakes.

Positive feedback is important, but hardly effective if it is not supported by giving students the possibility to try new strategies when they are stuck (Dweck, 2015). And this is exactly what we do by providing individualized feedback, including multiple representation scaffolding.

English Language Learners

To support emerging readers and learners of English, Happy Numbers uses clear, concise and definitive language. We also provide plenty of illustrations as visual support for text directions. Our audio feature allows students to hear any text read aloud in a clear, natural voice. A Spanish version of the system is also available.

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